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Mean Reversion in Net Discount Ratios: A Study in the Context of Fractionally Integrated Models

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Abstract

This article introduces a new alternative to the ongoing debate about stationarity and mean reversion of the net discount ratio. Modeling the net discount ratio as a fractionally integrated ($I(d)$) process, we apply recently developed frequency domain estimation procedures and find evidence that the net discount ratio is an $I(d)$ process with $1/2 \leq d < 1$. Although nonstationary, such series behave like stationary processes in one interesting respect; they are mean-reverting. We present results from a simulation experiment suggesting that the finding of a nonstationary, but mean-reverting net discount ratio generally supports the validity of current practice in estimating economic damages in personal injury litigation. Moreover, if recognized and accounted for, the presence of long memory in the net discount ratio even offers the potential to significantly improve forecasts of the present value of future earnings.

Introduction

There has been a long-running and, at times, heated debate in the literature on the estimation of damages in negligent injury and wrongful death/survivor claims. In these cases the injured party loses all or part of his or her ability to earn future wages. The objective of tort law is to place the injured party and dependents, if applicable, in the financial position they were in prior to the negligent injury. As a result, the amount of damages, both economic and noneconomic, that a person incurs as a result of a catastrophic injury can be substantial. The ability to accurately estimate such damages is more important than ever due to trends including increases in the cost of medical care, expansion of the types of damages claimed, and implementation of caps on noneconomic damages in healthcare professional liability insurance.

In this study, we are primarily concerned with the accurate estimation of lost future wages. In particular, we focus on the time series properties of the net discount ratio

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which is of central importance due to the need to factor in the time value of money and expected future wage growth when awarding damages. In practice, most courts award the amount of earnings that the injured person is expected to have earned after discounting to the present value.

As it is typically applied, the net discount rate method for estimating lost future wages involves determining the present value of the injured individual's earnings stream over a loss horizon. Specifically, if the loss horizon is T years, then the present value of lost wages, L, is computed as the present value of a T-year growing annuity. Thus

$$L = \frac{w_0(1+g)}{k-g}\left[1-\left(\frac{1+g}{1+k}\right)^T\right], \tag{1}$$

where w_0 is the injured individual's preinjury annual wage, g is the growth rate in earnings, and k is the nominal interest rate. Equation (1) is often expressed as

$$L = \frac{w_0}{NDR}\left[1-\left(\frac{1}{1+NDR}\right)^T\right], \tag{2}$$

where $NDR = (k-g)/(1+g)$ is called the net discount rate and is estimated from historical data by

$$NDR = \frac{1}{N}\sum_{i=1}^{N}\frac{k_i-g_i}{1+g_i}, \tag{3}$$

where g_i and k_i are past observations of the wage growth rate and the nominal interest rate, respectively. As in Braun, Lee, and Strazicich (2005), the net discount rate is sometimes defined as $1 + NDR = (1+k)/(1+g)$. Some studies, such as Haslag, Nieswiadomy, and Slottje (1991), Gamber and Sorensen (1994), and Hays et al. (2000) choose to examine the time series properties of the net discount *ratio*, (NDR^*), which is defined as $1/(1+NDR)$.[1] Since the time series properties of NDR, $1+NDR$, and NDR^* are practically identical, findings using one of the definitions will extend to all three. In this article, we choose to focus on the net discount ratio, NDR^*.

The stability of NDR^* over time is a crucial assumption underlying the net discount rate method as it is usually applied. In particular, the use of a constant NDR^* is only valid if NDR^* fluctuates around its unconditional mean. On the other hand, if NDR^* follows say, a unit root process, then there would be significant forecast error associated with assuming a constant NDR^*, and the resulting estimated damages would either be too high or too low.

[1] These studies consider the *real* net discount ratio $(1+g^*)/(1+r)$ where g^* is the growth rate in real earnings and r is the real interest rate. However, if both numerator and denominator are expressed in real terms, it is equivalent to the definition using nominal rates.

Review of the Literature

The evolution of the literature examining the time series properties of the net discount ratio closely follows advances in the time series econometrics literature. A few representative studies will illustrate this point. (See Table 1 for a comprehensive summary of the literature.)

In an early study, Lambrinos uses autocorrelation functions to suggest that nominal growth rates in wages and interest rates are nonstationary, while real rates are stationary. Lambrinos concludes that only real variables should be used in determining economic damages from lost future wages.

Haslag, Nieswiadomy, and Slottje (1991) use the Dickey and Fuller (1979) ADF test for a unit root and find that the growth rate of real wages is stationary, while real interest rates are not. However, they then suggest that the ratio of the two (the real net discount ratio) is stationary, using 3-month, 6-month, 1-year, and 3-year interest rates.

Gamber and Sorensen (1994) use a modification of the standard ADF unit root test suggested by Zivot and Andrews (1992). This test allows a one-time shift in the mean of the time series at an *a priori* unknown (endogenous) time point. Using 3-month, 6-month, 1-year, and 3-year interest rates, they find that the nonlinear [real] net discount ratio is best described as a stationary series around a one-time shift in its mean" (p. 510). Responding to criticisms of the original Dickey and Fuller (1979) ADF unit root test from Gamber and Sorensen, Haslag, Nieswiadomy, and Slottje (1994) apply the unit root tests of Phillips and Perron (1988) and suggest that significant nonlinearities are not present in the [real] net discount ratio and that the variable can be represented as a stationary time series" (p. 517).

Using data from the period 1964–1998, Hays et al. (2000) extend the debate begun by Gamber and Sorensen (1994) and Haslag, Nieswiadomy, and Slottje (1994). Using the Cochrane (1988) variance ratio test and the Campbell and Mankiw (1987) variance decomposition test, they suggest that the real net discount ratio follows a trend stationary process with mean-reverting properties. They then use the Lo (1991) modified rescaled range (R/S) test (with moving windows of 5, 10, and 20 years) to show that shorter-term windows (of up to about 20 years) are relatively sensitive to short-term mean shifts. They then argue that present value calculations for relatively longer periods (they say about 20 years) can be used with confidence.

Braun, Lee, and Strazicich (2005) challenge the finding that the net discount rate is mean-reverting. They use a Lagrange multiplier (LM) test developed in Lee and Strazicich (2003) that allows two (endogenous) structural breaks to test the nominal net discount rate. As argued in Braun, Lee, and Strazicich, the advantage of the LM test is that it is free of bias and spurious rejections in the presence of breaks under the [unit root] null (p. 6). Using 91-day, 3-year, and 10-year interest rates, they apply this test to data for the period 1948–1995. They find that the unit root null (allowing two structural breaks) cannot be rejected and suggest that the nominal net discount rate is nonstationary and mean averting. They confirm this finding with an examination of permanent and transitory components of nominal net discount rates using an unobserved components model.

TABLE 1
Previous Literature on the Time Series Properties of the Net Discount Ratio (Rate)

Study	Methodology	Treasury Maturities	Sample Period	Findings
Haslag, Nieswiadomy, and Slottje (1991)	Augmented Dickey-Fuller unit root tests	3 mo, 6 mo, 1 yr and 3 yr	1964:01–1989:04	Real NDR^* is stationary.
Haslag, Nieswiadomy, and Slottje (1994)	Phillips-Perron test and Stock-Watson q-test	3 mo, 6 mo, 1 yr and 3 yr	1964:01–1989:04	Real NDR^* is stationary.
Gamber and Sorensen (1994)	Augmented Dickey-Fuller unit root tests. Zivot and Andrews (ZA) methodology allowing for one structural break	3 mo, 6 mo, 1 yr and 3 yr	(1) 1964:01–1989:04 (2) 1964:04–1993:05	Real NDR^* is nonstationary but the source of the nonstationarity is a structural break between October 1979 and January 1980.
Johnson and Gelles (1996)	Graphical analysis, comparisons of means and variances	91 day, 3 year and 10 year annual yields	1953–1995 annual data	Structural break occurred around 1980. Yields were significantly different post-1980s than pre-1980s.
Horvath and Sattler (1997)	Dummy variable regression and Chow test of yield slopes	91 day, 3 year and 10 year annual yields	1953–1995 annual data	Found additional support for the Johnson and Gelles (1996) contention of a regime change occurring around 1980. Yields were significantly different post-1980s than pre-1980s.
Payne, Ewing, and Piette (1999b)	Augmented Dickey-Fuller unit root tests and Cochrane variance ratio tests	1 yr	1964:01–1996:10	The NDR is trend stationary.
Payne, Ewing, and Piette (1999a)	Perron's (1989) unit root tests with one structural break	3 mo, 6 mo, and 1 yr	1964:01–1996:10	The NDR is trend stationary after allowing for a structural break.
Hays et al. (2000)	Cochrane variance ratio test, Campbell-Mankiw decomposition test and R/S	3 mo, 6 mo, 1 yr and 3 yr	1964:02–1998:12	The NDR is trend stationary.
Sen et al. (2000)	Perron's test for a unit root with one time shift in mean	3 mo	1964:01–1999:03	Average NDR is trend stationary with a shift in mean in the third quarter of 1978.
Braun et al. (2005)	Two-break unit root test of Lee and Strazicich (2003)	91 day, 3 yr, 10 yr	1948–1995 annual data	The NDR is nonstationary even after accounting for two structural breaks

Clearly, the findings to date on the time series properties of the net discount ratio are mixed. Specifically, there seems to be some disagreement as to whether it is stationary and mean-reverting, or nonstationary and mean-averting. Using a newly developed methodology well suited to investigating just such a question, this study will seek to add some additional clarity to the debate.

There has been much recent progress in the time series literature on tests of nonstationarity. One of the most notable advances is the procedure for testing nonstationary hypotheses proposed by Robinson (1994). The Robinson (1994) parametric test uses a frequency domain test statistic derived by way of the score principle. This test allows considerable flexibility in the choice of null hypothesis, including integer or fractional roots of arbitrary order anywhere on the unit circle in the complex plane. Moreover, the test is asymptotically locally most powerful against fractional alternatives. In order to allow for weak parametric autocorrelation of the regression disturbances, Robinson suggests using the exponential spectrum model of Bloomfield (1973). The Bloomfield model provides a good approximation to stationary and invertible ARMA processes without having to estimate nuisance parameters for the order of autocorrelation such as in the popular ADF test and its variants.

The test of Robinson (1994) has been further developed and applied in several studies including Gil-Alana and Robinson (1997), which clearly demonstrate the generality and flexibility of Robinson's approach to testing nonstationary hypotheses. In this article, we will use the Robinson (1994) test with exponential spectrum model of Bloomfield (1973) to estimate the order of integration of the net discount ratio.

The remainder of this article is organized as follows. The section "Modeling the Net Discount Ratio as a Fractionally Integrated Process" reviews the definition of fractional integration and summarizes the properties of fractionally integrated time series that are important to the subsequent arguments. The section "Methodology" reviews the test of Robinson (1994) as it applies to estimating the order of integration of a fractionally integrated process and details the results of these tests for the order of integration of the net discount ratio. The final "Conclusions" section provides a brief summary of our research motivation and main findings.

MODELING THE NET DISCOUNT RATIO AS A FRACTIONALLY INTEGRATED PROCESS

In the $I(1)/I(0)$ paradigm, stationarity is equivalent to mean reversion. However, in general, a time series will exhibit mean reversion as a consequence of stationarity, although stationarity is not a necessary condition for mean reversion. One class of models encompassing both stationary and nonstationary mean-reverting series is the class of fractionally integrated processes.[2]

[2] It should be noted that even though fractionally integrated processes with d-values in $[1/2, 1)$ are often referred to as "mean-reverting" in the literature (cf. Robinson, 2003, p. 20), technically speaking such processes have no (asymptotic) sample mean. When describing $I(d)$ series with d-values in $[1/2, 1)$, "mean-reverting" refers to the behavior of a series with dissipating shocks, as opposed to a series with shocks that never decay such as a unit root process.

Definitions for Fractionally Integrated Models and Rationale for Their Use

A univariate series x_t is said to be integrated of order d ($x_t \sim I(d)$) if x_t has the representation

$$(1-L)^d x_t = u_t I\{t \geq 1\}, \quad t = 0, \pm 1, \ldots, \tag{4}$$

where L is the lag operator, u_t is a stationary process with $E[u_t] = 0$ for all t, and the spectral density of u_t is positive and continuous at the zero frequency.[3] More explicitly, by expanding $(1-L)^d$, Equation (4) can be rewritten as

$$\sum_{k=0}^{t} \frac{\Gamma(k-d)}{\Gamma(k+1)\Gamma(-d)} x_{t-k} = u_t I\{t \geq 1\}, \quad t = 0, \pm 1, \ldots,$$

where Γ is the Gamma function. If $x_t \sim I(d)$ for $d \notin \mathbb{Z}$, then x_t is said to be fractionally integrated. Such processes can be classified by order of integration, d, and the associated time series properties. The series is mean-reverting if and only if $d < 1$. Furthermore if $d < 1/2$, then the series is asymptotically stationary. A series that is integrated of order d with $d \in [1/2, 1)$ is nonstationary, but shocks eventually dissipate producing mean-reverting behavior.

The rationale for using fractionally integrated models follows from the fact that they offer greater generality and more flexibility compared to classical $I(1)/I(0)$ models. Starting with the path-breaking work of Dickey and Fuller (1979), much of the subsequent formal analysis in the mainstream time series econometrics literature has focused on unit root ($I(1)$) series. This emphasis on unit roots was due to the belief that many economic time series follow a random walk, a subcategory of a unit root process.

Shortly after the initial work by Dickey and Fuller Granger and Joyeux (1980), and Hosking (1981) suggested the potential usefulness of fractionally differenced models, and Geweke and Porter-Hudak (1983) developed a log-periodogram regression technique for estimating the order of fractional integration, commonly referred to as the GPH estimator. However, several studies have pointed out problems with the GPH estimator (see e.g., Agiakloglou, Newbold, and Wohar, (1979), 1993).

As shown by Kramer (1998) and others, the classic ADF unit root test is not consistent against fractional alternatives, except in special cases. This potential

[3] The model of fractional integration defined here is called "Type II" by Shimotsu and Phillips (2006), and it is also the model considered in Robinson (1994). The difference between "Type I" and "Type II" models of fractional integration is essentially in how the fractional differencing operator $(1-L)^d$ is defined. In the "Type I" definition, one inverts the operator $(1-L)^d$ and represents it as an infinite order moving average. Such a representation is possible provided that $d < 1/2$. In "Type II" fractional integration, the process is initialized at time $t = 0$ with all past shocks set to zero and the fractional differencing operator $(1-L)^d$ expressed as a finite order moving average. The advantage of this model of fractional integration is that it is valid for all values of d.

lack of sensitivity is troubling for any of the myriad of unit root tests. As these limitations of the $I(1)/I(0)$ paradigm became better understood, more focus shifted to the development of estimators for fractionally integrated models that improve on the performance of the Geweke and Porter-Hudak (1983) estimator.

Given the conflicting conclusions about the time series properties of the net discount ratio reached by previous studies that, for the most part, used classical unit root tests, it is reasonable to expect that a resolution may be found in the more general and flexible environment of fractional integration. Indeed, we will show that the time series properties of the net discount ratio can be adequately explained in the setting of fractionally integrated models.

METHODOLOGY

One may test the hypothesis H_0: $d = d_0$ in $(1 - L)^d x_t = u_t$ versus the alternatives H_a: $d < d_0$ or H_a: $d > d_0$ using the Robinson (1994) test. Robinson's test statistic, $\hat{r}$, is given by

$$\hat{r} = \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2},$$

with

$$\hat{\sigma}^2 = \min_{\tau\in\mathbb{R}} \sigma^2(\tau) = \min_{\tau\in\mathbb{R}} \frac{2\pi}{T} \sum_{j=1}^{T-1} \frac{I(u_t;\omega_j)}{h(\omega_j;\tau)}, \quad \hat{\tau} = \arg\min_{\tau\in\mathbb{R}} \sigma^2(\tau), \tag{5}$$

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\omega_j) \frac{I(u_t;\omega_j)}{h(\omega_j;\hat{\tau})}, \quad \psi(\omega_j) = \log\left[2\sin\left(\frac{\omega_j}{2}\right)\right],$$

$$\hat{\varepsilon}(\omega_j) = D_\tau[\log(h(\omega_j;\hat{\tau}))]$$

$$\hat{A} = \frac{2}{T}\left(\sum_{j=1}^{T-1}\psi(\omega_j)\psi(\omega_j)' - \sum_{j=1}^{T-1}\psi(\omega_j)\hat{\varepsilon}(\omega_j)'\left(\sum_{j=1}^{T-1}\hat{\varepsilon}(\omega_j)\hat{\varepsilon}(\omega_j)'\right)^{-1}\sum_{j=1}^{T-1}\hat{\varepsilon}(\omega_j)\psi(\omega_j)'\right),$$

where $I(\cdot;\omega)$ is the periodogram of its argument evaluated at ω, and $\omega_j = 2\pi j/T$ for $j = 1, \ldots, T - 1$, and h is defined in terms of the spectral density function.

Furthermore, x_t can be the errors in a multiple regression model. Specifically for a given series y_t, x_t can be the errors from the regression model

$$y_t = \beta' z_t + x_t, \quad t = 1, 2, \ldots,$$

where z_t is a vector of nonstochastic variables including such cases as $z_t = 1$ (intercept) and $z_t = (1, t)'$ (intercept and trend).

We take the exponential model of Bloomfield (1973) as our model for the spectrum of u_t. Specifically, we take h to be given by

$$h(\omega; \tau) = \exp\left\{2\sum_{r=1}^{m} \tau_r \cos(r\omega)\right\}. \tag{6}$$

The number, m, of summands in the exponential in (6) is called the order of the model. Bloomfield's motivation for using (6) was his observation that it was often possible to approximate the logarithm of an estimated spectral density by a truncated Fourier series. Since spectral densities are even functions, one need only consider the cosine terms of the series. Bloomfield's exponential spectrum and Robinson's frequency domain test statistic combine to yield particularly convenient tests from the computational perspective since the minimization in (5) can be carried out using Newton-type iterative methods.

Data

Following several previous studies, we construct NDR^* series using four different series for the nominal interest rate, k_t: the 3- and 6-month Treasury bill yield series plus the 1- and 3-year constant-maturity Treasury rate series. The monthly growth rates in *Average Hourly Earnings: Total Private Industries* monthly series (1964:1–2005:12) from the Bureau of Labor Statistics (BLS) is used as a proxy for g_t. We also study sector net discount ratios for different industries. We construct sector NDR^* series using the BLS standard of aggregating private NAICS industry sectors into 10 supersectors. Where applicable, all data have been seasonally adjusted since seasonality can potentially bias tests of persistence toward findings of long memory.

Results of Fractional Integration Analysis

We begin by conducting a fractional integration analysis on the individual series using the estimator from Robinson (1994) as in Gil-Alana and Robinson (1997). Specifically for each series y_t, we consider the regression models

$$y_t = \beta' z_t + x_t, \quad t = 1, 2, \ldots,$$

where z_t is a vector of nonstochastic variables. In this study, we take $z_t = (1, t)'$ in order to allow for intercepts and trends. Then we conduct one-sided tests of the null hypothesis H_0: $d = d_0$ in the model $(1 - L)^d x_t = u_t$, against the alternatives $d < d_0$ and $d > d_0$, assuming that the spectrum of the u_t series can be specified as Bloomfield's exponential model of order 2.[4] Finite sample critical values are reported in Gil-Alana (2000), although due to the sample size ($n = 504$), inference will be similar if we use asymptotic $N(0, 1)$ critical values. Significantly higher positive values of $\hat{r}$ are consistent with higher orders of integration ($d > d_0$), while significantly lower negative values suggest lower orders of integration ($d < d_0$).

[4] Bloomfield models with $m = 3$, 4, and 5 were also considered and found to produce similar results.

TABLE 2
Nominal Interest Rates

	0.70	0.75	0.80	0.85	0.90	0.95	1.00	1.05	1.10
3 mo.	2.467	**1.616***	**0.848***	**0.157***	**−0.463***	**−1.017***	**−1.517***	−1.966	−2.371
6 mo.	2.477	**1.610***	**0.828***	**0.126***	**−0.503***	**−1.069***	**−1.575***	−2.028	−2.437
1 yr.	2.830	1.918	**1.100***	**0.369***	**−0.283***	**−0.863***	**−1.381***	−1.844	−2.260
3 yr.	3.963	2.891	1.942	**1.103***	**0.364***	**−0.288***	**−0.862***	**−1.370***	−1.824

Notes: This table reports results of tests for order of fractional integration in monthly nominal interest rates (1964:1–2005:12). Values of Robinson's $\hat{r}$ statistic for each maturity and each value of d_0 are given.
Bold entries with * denote nonrejection values at the 5% significance level.

TABLE 3
Growth Rate of Nominal Wages

0.50	0.55	0.60	0.65	0.70	0.75	0.80
2.608	**1.425***	**0.853***	**0.019***	**−0.559***	**−1.444***	−1.992

Notes: This table reports results of tests for order of fractional integration in the monthly growth rate in nominal wages (1964:1–2005:12). Values of Robinson's $\hat{r}$ statistic for each value of d_0 are given.
Bold entries with * denote nonrejection values at the 5% significance level.

As summarized in Table 2, for both the 3-month and 6-month maturities, the null hypothesis H_0: $d = d_0$ is rejected at the 5 percent level with $d_0 = 1.05$ for lower alternatives and with $d_0 = 0.70$ for higher alternatives. For the 1-year maturity, the null hypothesis H_0: $d = d_0$ is rejected at the 5 percent level with $d_0 = 1.05$ for lower alternatives and with $d_0 = 0.75$ for higher alternatives. Finally, for the 3-year maturity, the null hypothesis H_0: $d = d_0$ is rejected at the 5 percent level with $d_0 = 1.10$ for lower alternatives and with $d_0 = 0.80$ for higher alternatives. Overall, a unit root is not rejected for any of the four series, although in each case the nonrejection interval intersects more of the mean-reverting range, $1/2 \leq d < 1$ than the non-mean-reverting range $d \geq 1$. There is also evidence that the order of integration is increasing in maturity as there is a rightward shift in the nonrejection region as maturity increases.

As summarized in Table 3, for the growth rate of nominal wages, g_t, the null hypothesis H_0: $d = d_0$ is rejected at the 5 percent level with $d_0 = 0.80$ for lower alternatives and with $d_0 = 0.50$ for higher alternatives. We conclude that g_t is mean-reverting with long memory and note that stationarity ($d < 1/2$) can be rejected at the 5 percent level.

Our findings are generally consistent with previous studies such as Haslag, Nieswiadomy, and Slottje (1991) that use standard unit root tests to reject the unit root null for the wage growth rate but not for interest rates. However, we emphasize that the Robinson (1994) test allows for much more precise inference since tests of a fractional null hypotheses can be directed against fractional alternatives. Indeed, we also reject stationarity ($d < 1/2$) for the wage growth rate but conclude that it is still

TABLE 4
Net Discount Ratio

	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
3 mo.	3.799	2.522	**1.258***	**0.416***	**−0.337***	**−1.099***	**−1.538***	−1.976	−2.314	−2.827	−3.103
6 mo.	3.716	1.966	**0.948***	**0.258***	**−0.674***	**−1.061***	−1.648	−1.988	−2.587	−2.907	−3.065
1 yr.	4.098	2.518	**1.305***	**0.200***	**−0.417***	**−0.914***	**−1.463***	−2.057	−2.453	−2.788	−3.058
3 yr.	4.356	2.515	**1.345***	**0.494***	**−0.096***	**−0.814***	**−1.336***	−2.036	−2.454	−2.683	−3.391

Notes: This table reports results of tests for order of fractional integration in the monthly net discount ratio (1964:2–2005:12). With x_t given as the regression residuals from $y_t = \beta_0 + \beta_1 t + x_t$, one-sided tests of the null hypothesis H_0: $d = d_0$ in the model $(1 - L)^d x_t = u_t$, against the alternatives $d < d_0$ and $d > d_0$ can be conducted. Values of Robinson's $\hat{r}$ statistic for each version of the net discount ratio and each value of d_0 are given.
Bold entries with * denote nonrejection values at the 5% significance level.

mean-reverting. The advantage of the flexibility of the Robinson (1994) test will also be apparent as we turn now to an examination of the net discount ratio.

Results of tests for order of fractional integration in the monthly net discount ratio constructed using each of the four interest rate series are reported in Table 4. In each case, the null hypothesis $H_0 : d = d_0$ with $d_0 = 0.85$ is rejected at the 5 percent level for lower alternatives. Using the 3-month and 6-month maturities for interest rates, $d_0 = 0.55$ is rejected at the 5 percent level for higher alternatives, while using the 1-year and 3-year maturities, $d_0 = 0.60$ is rejected at the 5 percent level for higher alternatives. Overall, these results suggest that the net discount ratio is an $I(d)$ series with $d \in (0.55, 0.85)$. In particular, stationarity as well as the unit root null are *both* rejected. Fractionally integrated series with orders of integration in the interval [1/2, 1) are not stationary, yet they are still mean-reverting. Such series exhibit long memory, yet shocks do eventually decay.

We then perform similar tests for order of fractional integration in the monthly net discount ratios for the individual economic sectors. The results are reported in Table 5. Results for four sectors, *Education and Health Services*, *Financial Activities*, *Professional and Business Services*, and *Other Services*, are qualitatively similar to the general results indicating orders of fractional integration in the interval [1/2, 1). Together, these four sectors account for about 34.4 percent of all U.S. employment. For five of the remaining six sectors, we cannot reject stationarity at the 5 percent level (with the exception of the 1-year and 3-year NDR^* for the *Manufacturing* sector), however in each case point estimates of d-values are greater than 1/2, suggesting that they are more likely nonstationary.[5] These five sectors together account for about 47.9 percent of all U.S. employment. Only in the case of *Natural Resources and Mining* do we reject $d_0 = 1/2$ in favor of lower alternatives, indicating that NDR^* for this sector is stationary (although we still find some long memory since we can also reject $d_0 = 0$ for higher alternatives).

[5] Since the Robinson (1994) test is based on the Whittle function (an approximation to the likelihood function), the value of d_0 that produces the lowest test statistic in absolute value is an approximation to the maximum likelihood estimate.

TABLE 5
Sector Net Discount Ratio

Sector	% All Emp.	3 Months	6 Months	1 Year	3 Years
Construction	5.4	[0.41 (0.51) 0.68]	[0.41 (0.50) 0.68]	[0.44 (0.53) 0.69]	[0.46 (0.56) 0.72]
Education and Health Services	12.4	[0.57 (0.66) 0.80]	[0.56 (0.65) 0.78]	[0.57 (0.67) 0.80]	[0.57 (0.65) 0.78]
Financial Activities	6.1	[0.51 (0.65) 0.84]	[0.50 (0.63) 0.82]	[0.52 (0.65) 0.84]	[0.50 (0.63) 0.81]
Information	2.4	[0.47 (0.57) 0.72]	[0.47 (0.56) 0.72]	[0.49 (0.59) 0.73]	[0.49 (0.58) 0.73]
Leisure and Hospitality	9.6	[0.44 (0.52) 0.62]	[0.44 (0.51) 0.61]	[0.46 (0.53) 0.63]	[0.47 (0.54) 0.63]
Manufacturing	11.0	[0.49 (0.57) 0.68]	[0.49 (0.57) 0.67]	[0.50 (0.58) 0.69]	[0.51 (0.59) 0.69]
Natural Resources and Mining	8.6	[0.31 (0.38) 0.47]	[0.30 (0.37) 0.46]	[0.32 (0.39) 0.48]	[0.33 (0.39) 0.48]
Other Services	3.3	[0.56 (0.70) 0.87]	[0.56 (0.68) 0.86]	[0.56 (0.69) 0.86]	[0.56 (0.70) 0.86]
Professional and Business Services	12.6	[0.61 (0.71) 0.86]	[0.60 (0.70) 0.84]	[0.62 (0.72) 0.86]	[0.64 (0.73) 0.87]
Trade, Transportation & Utilities	19.5	[0.43 (0.52) 0.63]	[0.43 (0.51) 0.62]	[0.44 (0.52) 0.62]	[0.44 (0.52) 0.62]

Note: This table reports results of tests for order of fractional integration in the monthly net discount ratio (1964:2–2005:12) for the 10 different BLS nongovernment supersectors. With x_t given as the regression residuals from $y_t = \beta_0 + \beta_1 t + x_t$, one-sided tests of the null hypothesis $H_0 : d = d_0$ in the model $(1 - L)^d x_t = u_t$, against the alternatives $d < d_0$ and $d > d_0$ are conducted. Using the format $[d_{0,l}(d_{0,p})\, d_{0,u}]$, we report $d_{0,l}$, the lowest nonrejection value at the 5% level for the alternative $d < d_0$, $d_{0,p}$, the point estimate of the d-value, and, $d_{0,u}$, the greatest non -rejection value at the 5% level for the alternative $d > d_0$. We also report the percentage of all employment in the U.S. economy accounted for by each supersector.

It is interesting to note that estimated d-values for NDR^*s in sectors corresponding to noncyclical industries are generally greater than those corresponding to cyclical industries. We find the highest d-values in the noncyclical service sectors, followed closely by *Financial Activities*, which consists largely of financial services occupations. The more cyclical sectors like *Manufacturing*, *Leisure and Hospitality* and *Construction* have lower d-values. We find the lowest d-values of all in the cyclical *Natural Resources and Mining* sector. Although a complete economic explanation for the differences in d-values for NDR^*s across sectors is beyond the scope of this article, it is at least tempting to project that in the United States, the continued gradual shift from a manufacturing economy to a service economy may result in a gradual increase in d-values for the economy-wide NDR^*.

Implications of Fractional Integration for Forecasting

We have presented evidence that the economy-wide as well as the sector NDR^* are long memory processes that are mean-reverting. In most cases, our results suggest that the NDR^* is nonstationary. If shocked away from its historical average, a mean-reverting but nonstationary NDR^* will eventually revisit this level. However, there is a strong possibility that the time path away from this level will be much longer compared to the case of a stationary NDR^*. The practice of forecasting the present value of future earnings using a constant NDR^* will be valid provided that the NDR^* is "stable." Yet, the literature has typically interpreted "stable" to mean stationary. In assessing the significance of our findings to current practice, the basic question becomes: Is a mean-reverting but nonstationary NDR^* stable enough? But another, perhaps more important, question is: can the presence of long memory be exploited to improve forecasts of future earnings? When working with long memory processes, forecasts that essentially rely on point estimates of the mean will generally be inferior to forecasts that place more emphasis on recent observations. In fact, Granger and Joyeux (1980) establish that long memory actually improves N-step forecastability. So, the finding of long memory in NDR^* suggests that the net discount rate method can be improved provided that the long-memory is recognized and accounted for in forecasts.

To illustrate these forecasting implications, we conduct a simulation experiment. Suppose that the current (year 0) value of NDR^* is one standard deviation greater than its value in the previous year. Considering loss horizons of 1–50 years, we compare the amounts of awards based on the historical sample mean NDR^*, with a simulation estimate of the present value of lost future wages assuming NDR^* is fractionally integrated with d-values of 0.7, 0.5, and 0.3. In addition to the overall economy-wide NDR^*, these orders of integration also cover the different possible qualitative behaviors of NDR^* for the sectors according to the results of the previous section. We calibrate the models using the actual NDR^* sample mean of 0.982218 and sample standard deviation of 0.033089 for our sample period (1964–2005). We repeat the experiment introducing a negative one standard deviation shock at year 0.

For each loss horizon T, we generate $1000 + 200 \times T$ realizations of a white noise series $\varepsilon_{T,n,t}$, of length $50 + T$, with $-49 \leq t \leq T$.[6] Then for each n, set $\varepsilon_{T,n,0} = 1$ ($\varepsilon_{T,n,0} = -1$)

[6] Generating 200 additional paths per year serves to reduce the variance in our final estimates.

to produce the 1 (-1) standard deviation shock at year 0 (the first 49 values serve to initialize the series). Next, to each white noise series apply the filter $(1-L)^{-d}$ to produce $(1-L)^{-d}\,\varepsilon_{T,n,t}$. We then scale $(1-L)^{-d}\,\varepsilon_{T,n,t}$ by the historical sample standard deviation of 0.033089 and shift the series so that its value at $t = 0$ is equal to the historical sample mean of 0.982218. The resulting series $y_{T,n,t}$ for $t > 0$ is fractionally integrated of order d.

As summarized in Table 6, in both the positive and negative year 0 shock cases, differences in the present values of future earnings (PVFE) computed using a constant NDR^* and using a simulated fractionally integrated NDR^* are economically significant in most cases. Across all d-values, the *Dollar Differences* in PVFE are increasing in *Years* in the positive shock cases, and decreasing in *Years* through around the 10-year loss horizon in the negative shock case. The *% Differences* are generally increasing in *Years* in the positive shock case, while initially decreasing but then increasing with the switch occurring between the 10- and 20-year loss horizons. It is worth noting that the asymmetries in *% Differences* (as functions of *Years*) between the positive and negative shock simulation results are to be expected due to the effects of compounding and long memory in fractionally integrated processes.[7]

Overall, the simulation study demonstrates that if the NDR^* is actually fractionally integrated with a d-value of at least 0.3, then calculating PVFE assuming a constant NDR^* will tend to understate lost earnings following positive shocks and overstate lost earnings following negative shocks. These errors are generally magnified for a given loss horizon as the d-value increases. Furthermore, the simulations reveal that the effects on PVFE estimates from positive and negative shocks in the prior year are not symmetric beyond loss horizons of about 5 years. Differences, in both dollar and percentage terms, are greater following positive shocks, and the differences are magnified at longer time horizons. Following negative shocks, differences stabilize and eventually even begin to decline in magnitude. As a practical matter, we would argue that the simulations generally support the validity of current practice in estimating lost future wages, especially in times following negative shocks in NDR^*. Moreover, our results clearly indicate that more accurate estimates of lost earnings can be obtained using simulation methods that take into account the long memory present in NDR^* and incorporate recent shocks into PVFE estimates.

CONCLUSIONS

As it is usually practiced in wrongful death and personal injury litigation, estimation of the present value of expected future losses tacitly presupposes that net discount ratios are mean-reverting. However, there is considerable disagreement in the literature regarding the time series properties of net discount ratios. Different studies

[7] Even though the simulated process begins at a level below the constant (0.9822) process in the negative shock case, any excursions it makes above 1 will tend to persist due to the long memory and the effects will be compounded. For instance, an NDR^* of 1.10 and 1.05 in two successive years will increase the present value of future earnings by 15.5 percent ($1.10 \times 1.05 = 1.155$). On the other hand, an NDR^* of 0.90 and 0.95 in two successive years will decrease the present value of future earnings by only 14.5 percent. Since the probability that a path spends time above 1 increases as the loss horizon increases, this effect only becomes apparent in the simulations at longer horizons.

TABLE 6
Simulation

Panel A

Years	PVFE (NDR^* const.)	$d = 0.7$ Simulated PVFE	\$ Diff.	% Diff.	$d = 0.5$ Simulated PVFE	\$ Diff.	% Diff.	$d = 0.3$ Simulated PVFE	\$ Diff.	% Diff.
1	49,110	49,867	756	1.54	49,813	702	1.43	49,612	501	1.02
2	97,349	99,497	2,149	2.21	99,052	1,704	1.75	98,366	1,018	1.05
3	144,728	148,613	3,885	2.68	147,721	2,993	2.07	146,722	1,994	1.38
4	191,266	196,855	5,590	2.92	196,378	5,113	2.67	194,038	2,773	1.45
5	236,976	246,194	9,218	3.89	243,702	6,727	2.84	240,767	3,792	1.60
6	281,873	291,799	9,926	3.52	290,173	8,300	2.94	287,922	6,049	2.15
7	325,971	341,183	15,212	4.67	337,792	11,821	3.63	331,959	5,988	1.84
8	369,286	386,322	17,037	4.61	383,723	14,438	3.91	378,394	9,109	2.47
9	411,830	432,796	20,966	5.09	428,942	17,113	4.16	421,293	9,463	2.30
10	453,618	481,025	27,407	6.04	475,055	21,438	4.73	464,765	11,147	2.46
15	651,671	700,598	48,926	7.51	684,677	33,006	5.06	669,185	17,514	2.69
20	832,731	910,171	77,440	9.30	884,181	51,451	6.18	859,780	27,049	3.25
25	998,255	1,110,413	112,158	11.24	1,073,715	75,460	7.56	1,031,829	33,575	3.36
30	1,149,576	1,291,452	141,875	12.34	1,243,449	93,872	8.17	1,196,117	46,541	4.05
35	1,287,914	1,462,631	174,717	13.57	1,401,188	113,274	8.80	1,339,724	51,810	4.02
40	1,414,382	1,624,016	209,634	14.82	1,549,303	134,921	9.54	1,470,792	56,410	3.99
45	1,529,999	1,782,521	252,523	16.50	1,686,230	156,232	10.21	1,595,845	65,847	4.30
50	1,635,694	1,923,113	287,419	17.57	1,809,235	173,541	10.61	1,711,445	75,751	4.63

(continued)

TABLE 6
(Continued)

Panel B

Years	PVFE (NDR^* const.)	$d = 0.7$ Simulated PVFE	\$ Diff.	% Diff.	$d = 0.5$ Simulated PVFE	\$ Diff.	% Diff.	$d = 0.3$ Simulated PVFE	\$ Diff.	% Diff.
1	49,110	48,442	−669	−1.36	48,439	−672	−1.37	48,700	−411	−0.84
2	97,349	95,368	−1,981	−2.03	95,660	−1,688	−1.73	96,292	−1,057	−1.09
3	144,728	141,002	−3,726	−2.57	142,151	−2,577	−1.78	142,838	−1,891	−1.31
4	191,266	185,916	−5,350	−2.80	186,920	−4,346	−2.27	188,274	−2,992	−1.56
5	236,976	229,673	−7,302	−3.08	231,396	−5,579	−2.35	232,644	−4,332	−1.83
6	281,873	273,175	−8,697	−3.09	274,538	−7,334	−2.60	276,934	−4,938	−1.75
7	325,971	315,276	−10,695	−3.28	317,492	−8,479	−2.60	320,665	−5,307	−1.63
8	369,286	357,408	−11,878	−3.22	359,122	−10,164	−2.75	362,925	−6,361	−1.72
9	411,830	397,543	−14,287	−3.47	400,277	−11,552	−2.81	405,385	−6,445	−1.56
10	453,618	436,409	−17,208	−3.79	440,141	−13,477	−2.97	444,861	−8,756	−1.93
15	651,671	630,400	−21,271	−3.26	632,788	−18,883	−2.90	639,500	−12,171	−1.87
20	832,731	802,366	−30,365	−3.65	805,945	−26,786	−3.22	819,787	−12,944	−1.55
25	998,255	970,947	−27,308	−2.74	971,189	−27,066	−2.71	984,542	−13,713	−1.37
30	1,149,576	1,123,701	−25,875	−2.25	1,116,273	−33,303	−2.90	1,128,633	−20,944	−1.82
35	1,287,914	1,259,800	−28,114	−2.18	1,262,773	−25,141	−1.95	1,268,845	−19,069	−1.48
40	1,414,382	1,387,369	−27,013	−1.91	1,384,790	−29,592	−2.09	1,394,384	−19,998	−1.41
45	1,529,999	1,504,233	−25,765	−1.68	1,500,349	−29,649	−1.94	1,510,886	−19,111	−1.25
50	1,635,694	1,632,031	−3,663	−0.22	1,615,877	−19,817	−1.21	1,618,352	−17,342	−1.06

Notes: This table reports results of a simulation experiment illustrating the consequences of using a constant NDR^* to estimate lost future earnings if NDR^* is actually fractionally integrated. For each loss horizon T, we first calculate the present value of future earnings (PVFE) given an initial annual income of \$50,000 based on the historical sample mean NDR^* of 0.9822. Then we simulate NDR^* by generating $1000 + 200 \times T$ sample paths of a fractionally integrated process with d-values of 0.70, 0.50, and 0.30, and calculate the simulated PVFE. We first consider the case (reported in Panel A) with each sample path having a one standard deviation positive shock at time 0 and then we consider the case (reported in Panel B) with each sample path having a one standard deviation negative shock at time 0. We calibrate the simulated series using the actual NDR^* sample mean of 0.982218 and sample standard deviation of 0.033089 for our sample period (1964–2005).

(summarized in Table 1) have concluded that net discount ratios are stationary, nonstationary, stationary around a structural break, and nonstationary after accounting for two structural breaks. In light of the evidence presented in this article that net discount ratios are fractionally integrated, it becomes clear why previous studies using the $I(1)/I(0)$ paradigm often produced conflicting results. If the net discount ratio is actually $I(d)$ with $1/2 \leq d < 1$ then conventional unit root tests may erroneously suggest stationary, especially if the d-value is close to $1/2$.

By examining the net discount ratio in the setting of fractionally integrated time series, this article provides an explanation for the conflicting results previously reported in the literature. The finding of this study that, although nonstationary, net discount ratios are still mean-reverting generally supports the validity of current practice in estimating lost future wages. However, we demonstrate that more accurate estimates of lost earnings can be obtained by simulations that take long memory into account and accordingly incorporate the actual historical time path of the net discount ratio into the estimates.

For future research it would be interesting to apply the Robinson–Bloomfield methodology in a test that also estimates an endogenous structural break, and allows for changes in d-values at the break time. However, such a test has not yet appeared in the time series literature.

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